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# A DIRECT VERSION OF THE METHOD OF BOUNDARY INTEGRAL EQUATIONS IN THE THEORY OF ELASTICITY†

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A version of the direct method of boundary integral equations is considered which leads to systems of singular integral equations of the second kind for solving all the main types of boundary value problems of the theory of elasticity. Media with an arbitrary elastic anisotropy are discussed.

SINGULAR integral equations of the second kind in the direct method of boundary integral equations are obtained when solving the second boundary value problem with surface stresses specified on the boundary $\partial\Omega$ of the main region, if we write the Somigliana identity for $\partial\Omega$ [1]

$$(\tfrac{1}{2} + \mathbf{S}^{t})(\mathbf{u}_0)(\mathbf{x}') = \int_{\partial\Omega} \mathbf{t}_0(\mathbf{y}') \cdot \mathbf{E}(\mathbf{x}' - \mathbf{y}')\, dy' \tag{0.1}$$

Here $\mathbf{u}_0$ and $\mathbf{t}_0$ are the values of the surface strains and stresses, respectively, $\mathbf{E}$ is the fundamental Kelvin solution (the Kelvin–Boussinesq solution for the plane case), $\mathbf{I}$ is the unit diagonal matrix and $\mathbf{S}$ is the matrix of the singular operator obtained by contracting the potential of a double layer on $\partial\Omega$. In the case of the first boundary value problem relation (0.1) leads to an integral equation of the first kind (in $\mathbf{t}_0$) with a completely continuous kernel $\mathbf{E}$. Taking into account the fact that this problem is ill-posed, it is necessary to use regularization methods to obtain stable solutions numerically. Similar problems arise when solving mixed boundary value problems. Direct versions of the method of boundary integral equations are also possible when solving plane problems of the theory of elasticity using complex potentials [2].

To obtain equations of the second kind in direct versions of boundary integral equations when solving the first boundary value problem it is natural to act with the stress operator on both sides of the Somigliana identity. Then

$$(\tfrac{1}{2} + \mathbf{S}^{*})(\mathbf{t}_0)(\mathbf{x}') = \mathbf{G}_0(\mathbf{u}_0)(\mathbf{x}') \tag{0.2}$$

$$\mathbf{G}_0(\mathbf{u}_0)(\mathbf{x}') = \lim_{x'' \to x'} T(\nu_{x'}, \partial_{x''}) \int_{\partial\Omega} \mathbf{u}_0(\mathbf{y}') \cdot \mathbf{T}(\nu_{y'}, \partial_{y'})\, \mathbf{E}(\mathbf{x}' - \mathbf{y}')\, dy' \tag{0.3}$$

The limits on the left-hand side of (0.3) are calculated along non-tangential directions to $\partial\Omega$. Equation (0.2) is an equation of the second kind in $\mathbf{t}_0$, but the operator $\mathbf{G}_0$ turns out to be supersingular. Similar supersingular operators also arise when solving mixed boundary value problems. The properties of these operators have not yet been investigated from the computational point of view [3].

Below we develop a version of the direct method which leads to equations of the second kind similar to (0.2) for all main types of boundary value problems of the theory of elasticity. It is shown that the operator $\mathbf{G}_0$ can be represented in the form of a composition of an integral operator with a weak singularity and a Beltrami–Laplace operator. This representation enables us to eliminate the neighbourhoods $\omega_\epsilon$ of the points containing non-integrable components of the supersingular integrals with an error $O[\mathrm{mes}(\omega_\epsilon)]$. This in turn enables us to extend existing software intended for evaluating singular integrals and integrals with a weak singularity to supersingular integrals of the form considered.

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## 1. FUNDAMENTAL RELATIONS

Consider a uniform anisotropic elastic medium, the equilibrium equations of which have the following form

$$\mathbf{A}(\partial_x)\,\mathbf{u} \equiv -\operatorname{div}\mathbf{C}\cdot\cdot(\nabla\mathbf{u}) \tag{1.1}$$

where $\mathbf{u}$ is the displacement vector and $\mathbf{C}$ is a fourth-order strictly elliptic elasticity tensor

$$(\eta\otimes\xi)\cdot\cdot\mathbf{C}\cdot\cdot(\xi\otimes\eta)>0,\quad \forall\ \eta,\xi\in R^3,\quad \eta,\xi\neq 0 \tag{1.2}$$

We will assume that the medium investigated is hyperelastic: this leads to symmetry of the tensor $\mathbf{C}$ with respect to the outer pairs of indices: $C^{mnij}=C^{ijmn}$. Condition (1.2) ensures that the matrix symbol $\mathbf{A}^\vee$

$$\mathbf{A}^\vee(\xi)=(2\pi)^2\xi\cdot\mathbf{C}\cdot\xi \tag{1.3}$$

obtained by applying a Fourier transformation

$$f^\vee(\xi)=\int_{R^3} f(x)\exp(-2\pi i\,\mathbf{x}\cdot\xi)\,dx,\quad f\in L^2(R^3)$$

to Eq. (1.1), is elliptic.

Using the symbol $\mathbf{A}^\vee$ it is easy to construct the inverse symbol $\mathbf{E}^\vee$, which is the Fourier-transformed fundamental solution

$$\mathbf{E}^\vee(\xi)=\mathbf{A}_0^\vee(\xi)/\det\mathbf{A}^\vee(\xi) \tag{1.4}$$

where $\mathbf{A}_0^\vee$ is the matrix of the cofactors of the symbol $\mathbf{A}^\vee$. Formula (1.4) shows that the symbol $\mathbf{E}^\vee$ is elliptic, real-analytic everywhere in $R^3\backslash 0$ and homogeneous with respect to $|\xi|$ of degree $-2$. A Fourier inversion of expression (1.4) in the general case of anisotropy can only be carried out numerically [4].

Support $\Omega$ is a bounded singly-connected region in $R^3$ with boundary $\partial\Omega$, which is an embedded compact $C^{m,\alpha}$-submanifold with $m\geq 1$, $\alpha>0$ in $R^3$. On the surface $\partial\Omega$ we are given the operator of the boundary conditions

$$\begin{aligned}&\mathbf{B}(\nu,\partial_x)\,\mathbf{u}\equiv(\mathbf{M}\cdot\mathbf{u}+\mathbf{N}\cdot\mathbf{T}(\nu,\partial_x)\,\mathbf{u})|_{\partial\Omega}=\mathbf{g}\\&\mathbf{T}(\nu,\partial_x)\,\mathbf{u}\equiv\nu\cdot\mathbf{C}\cdot\operatorname{sym}(\nabla\mathbf{u})\end{aligned} \tag{1.5}$$

where $\mathbf{M}$ and $\mathbf{N}$ are square matrices, $\mathbf{T}$ is the surface-stress operator and $\nu$ is the vector of the unit normal to $\partial\Omega$.

The operator $\mathbf{B}$ enables us to describe the different types of boundary conditions in the theory of elasticity by a single analytic expression. In particular, when $\mathbf{M}=\mathbf{I}$ and $\mathbf{N}=0$, where $\mathbf{I}$ is the unit diagonal matrix, this condition is the first boundary value problem; when $\mathbf{M}=0$ and $\mathbf{N}=\mathbf{I}$, this is the condition of the second boundary value problem; when $\mathbf{M}=\nu\otimes\nu$, $\mathbf{N}=\mathbf{I}-\nu\otimes\nu$ this is the condition of the third boundary value problem (the Hadamard problem), when the normal component of the displacement vector and the shear stresses are given on the boundary; when $\mathbf{M}=\mathbf{I}-\nu\otimes\nu$, $\mathbf{N}=\nu\otimes\nu$ we obtain the fourth boundary value problem. Other types of boundary conditions can be specified in a similar way.

## 2. BOUNDARY OPERATORS OF THE DIRECT METHOD

We will introduce the following factors

$$\mathbf{g}=\mathbf{M}\cdot\mathbf{u}_0+\mathbf{N}\cdot\mathbf{t}_0,\qquad \mathbf{f}=\mathbf{N}\cdot\mathbf{u}_0+\mathbf{M}\cdot\mathbf{t}_0 \tag{2.1}$$

It is obvious that for boundary value problems 1–4 the vectors $\mathbf{f}$ and $\mathbf{g}$ define the known and unknown vector densities on $\partial\Omega$, respectively.

By writing the Somigliana identity for points of the boundary surface $\partial\Omega$ and calculating the stresses on the boundary we obtain the following analogues of Eqs (0.1) and (0.2) for solving boundary value problems 1–4

$$\mathbf{K}(\mathbf{f})(\mathbf{x}') = \mathbf{G}(\mathbf{g})(\mathbf{x}') \tag{2.2}$$

$$\mathbf{K} = (\mathbf{I}/2 + \mathbf{S}^t)\cdot\mathbf{N} + (\mathbf{I}/2 + \mathbf{S}^*)\cdot\mathbf{M} = \mathbf{I}/2 + \mathbf{S}^t\cdot\mathbf{N} + \mathbf{S}^*\cdot\mathbf{M} \tag{2.3}$$

where **K** is the singular matrix operator. The kernel of the integro-differential operator **G** has the form

$$\mathbf{G}(\mathbf{x}', \mathbf{y}') = \mathbf{E}(\mathbf{x}' - \mathbf{y}')\cdot\mathbf{N} + \mathbf{G}_0(\mathbf{x}', \mathbf{y}')\cdot\mathbf{M} \tag{2.4}$$

In turn, the kernel of the supersingular operator $\mathbf{G}_0$ is defined by (0.3).

An analysis of relation (2.3) shows that the following holds.

*Proposition 1.* The operator **K** is a standard matrix pseudo-differential operator of class $S^\circ$ on $\partial\Omega$.

*Definition.* The spectrum of the operator **X** will be called a set of (complex) numbers $\lambda$, for which the operator $\lambda\mathbf{I} - \mathbf{X}$ is not invertible in the class of continuous endomorphisms which act in the corresponding functional space.

We will denote by $H^s(\partial\Omega, R^3)$ the Sobolev–Slobodetskii space of index $s \geqslant 0$. If $\partial\Omega$ is a manifold of class $C^{m,\alpha}$ and $s < 2m + \alpha$, the spaces $H^s$ are correctly defined on $\partial\Omega$. Henceforth, this condition, imposed on the index $s$, will be assumed to be satisfied.

*Lemma 1.* (A) The spectrum of the operator **S** is discrete. (B) The spectrum of **S** lies in the circle $|\lambda| \leqslant 1/2$. (C) The point $\lambda = -1/2$ belongs to the spectrum $S$ and is a simple pole of the resolvent. (D) The spectral subspace $\mathbf{E}_{-1/2}$ is six-dimensional and contains a contraction on $\partial\Omega$ of the rigid displacements: $\mathbf{c} + \mathbf{W}\cdot\mathbf{x}'$, where **W** is an arbitrary skew-symmetric tensor.

The lemma has been proved both in the isotropic case [5, 6] and in the anisotropic case [7].

The following proposition follows directly from the assertion of Lemma 1(C).

*Proposition 2.* The operator **K** is invertible in the factor-space $H^s(\partial\Omega, R^3)\backslash\mathbf{E}_{-1/2}$.

Hence, in $H^s(\partial\Omega, R^3)\backslash\mathbf{E}_{-1/2}$ the equation of the kind (2.2) is uniquely solvable

$$\mathbf{f}(\mathbf{x}') = \mathbf{K}^{-1}\circ\mathbf{G}(\mathbf{g})(\mathbf{x}') \tag{2.5}$$

Here it is assumed that the right-hand side $\mathbf{G}(\mathbf{g})(\mathbf{x}')$ belongs to the stated factor-space.

The inverse operator $\mathbf{K}^{-1}$ can be constructed using a Neumann series

$$\mathbf{K}^{-1} = 2\sum_{n=0}^{\infty}(-2\mathbf{S}^t\cdot\mathbf{N} - 2\mathbf{S}^*\cdot\mathbf{M})^n \tag{2.6}$$

In view of the statements of Lemmas 1(B) and (C), the Neumann series (2.6) is absolutely convergent in $H^s(\partial\Omega, R^3)\backslash\mathbf{E}_{-1/2}$. A similar method of constructing the inverse operator in the anisotropic case for boundary value problems 1 and 2 has been indicated previously [8].

*Note 1.* When solving the internal and external boundary value problems 1–4 by direct boundary integral equation methods [5, 6], the corresponding root spaces turn out to be different, and generally speaking, distinguishable from $\mathbf{E}_{-1/2}$. This is explained by the fact that in the above methods the boundary operator may also contain operators of the form $-\mathbf{I}/2 + \mathbf{S}$, where the point $\lambda = 1/2$ does not belong to the spectrum **S**.

## 3. THE PROPERTIES OF THE OPERATOR $\mathbf{G}_0$

The main result of this section is proving the possibility of eliminating regions containing non-integrable singularities of supersingular integrals.

The direct use of (0.3) for a supersingular kernel $\mathbf{G}_0$ is inconvenient since it requires us to calculate the limits in non-tangential directions. For the purposes of this section, more convenient formulae are obtained if we transfer to an investigation of the principal symbol $\mathbf{G}_0^{\sim}$

$$\mathbf{G}_0^{\sim}(\mathbf{x}',\xi')=\lim_{x''\to\pm 0}\int \mathbf{T}^{\vee}(\nu_{x'},\xi)\cdot\mathbf{E}^{\vee}(\xi)\cdot\mathbf{T}^{\vee t}(\nu_{x'},\xi)\exp(-2\pi i\xi''x'')\,dx''$$

$$\xi'=\xi-(\xi\cdot\nu_{x'})\nu_{x'},\quad \xi''=\xi\cdot\nu_{x'},\quad \mathbf{x}'\in\partial\Omega \tag{3.1}$$

The sign $\sim$ here and henceforth denotes a Fourier transformation with respect to the variable lying in the cotangent stratification $T^*\partial\Omega$. Henceforth the dependence on $x'$ will be omitted in the notation of the symbol $\mathbf{G}_0^{\sim}$.

The improper integral in (3.1) can be represented in a simpler form if we use the obvious equations

$$\mathbf{T}^{\vee}(\nu,\xi)=-\frac{A^{\vee}(\xi)-(2\pi)^2\xi'\cdot\mathbf{C}\cdot\xi}{2\pi i\xi''},\quad \mathbf{T}^{\vee t}(\nu,\xi)=-\frac{A^{\vee}(\xi)-(2\pi)^2\xi\cdot\mathbf{C}\cdot\xi'}{2\pi i\xi''} \tag{3.2}$$

and bear in mind that $\mathbf{A}^{\vee}(\xi)\circ\mathbf{E}^{\vee}(\xi)=\mathbf{I}$. Taking relations (3.2) into account we have

$$\begin{aligned}\mathbf{G}_0^{\sim}(\xi')=&-\mathrm{sym}[(2\pi i)(\xi'\cdot\mathbf{C}\cdot\xi')\cdot H_{\xi''}\mathbf{E}^{\vee}(\xi',0)\cdot(\xi'\cdot\mathbf{C}\cdot\nu)+\\&+(2\pi i)^2(\xi'\cdot\mathbf{C}\cdot\nu)\cdot\mathbf{E}^{\sim}(\xi',0)\cdot(\xi'\cdot\mathbf{C}\cdot\nu)+(2\pi i)^2(\xi'\cdot\mathbf{C}\cdot\xi')\cdot\mathbf{E}^{\sim}(\xi',0)\cdot(\nu\cdot\mathbf{C}\cdot\nu)+\\&+(2\pi i)(\xi'\cdot\mathbf{C}\cdot\nu)\cdot\partial_{x''}\mathbf{E}^{\sim}(\xi',0)\cdot(\nu\cdot\mathbf{C}\cdot\nu)]\end{aligned} \tag{3.3}$$

$$H_{\xi''}\mathbf{E}^{\vee}(\xi',0)=\mathrm{V.P.}\int_{-\infty}^{\infty}\frac{\mathbf{E}^{\vee}(\xi)}{2\pi i\xi''}\,d\xi'',\quad \mathbf{E}^{\sim}(\xi',0)=\int_{-\infty}^{\infty}\mathbf{E}^{\vee}(\xi)\,d\xi'' \tag{3.4}$$

$$\partial_{x''}\mathbf{E}^{\sim}(\xi',0)=\mathrm{V.P.}\int_{-\infty}^{\infty}2\pi i\mathbf{E}^{\vee}(\xi)\xi''d\xi''$$

Here $H_{\xi''}\mathbf{E}^{\vee}(\xi',0)$ is the value at zero of the Hilbert transformation of $\mathbf{E}^{\vee}(\xi)$ with respect to $\xi''$, while $\mathbf{E}^{\sim}(\xi',0)$ is the value of the partial Fourier transformation with respect to $\mathbf{x}'$ at $\mathbf{x}''=0$. We can determine $\partial_{x''}\mathbf{E}^{\sim}(\xi',0)$ in a similar way.

It is important that the integrals (3.4) are correctly defined for any $\xi'\neq 0$ since the symbol $\mathbf{E}^{\vee}$, considered as a function of a single parameter $\xi''$, is infinitely differentiable and belongs to the class $L^p, p\geq 1$.

A direct analysis of (3.3) shows that the following assertion holds.

*Proposition 3.* (A) The matric symbol $\mathbf{G}_0^{\sim}$ is a symbol of the class $S^1$ (positively homogeneous of degree 1). (B) The symbol $\mathbf{G}_0^{\sim}$ is positively semi-defined for any $\xi'\neq 0$

$$\mathbf{a}\cdot\mathbf{G}^{\sim}(\xi')\cdot\mathbf{a}\geq 0,\quad \forall\,\mathbf{a}\in R^3,\quad \mathbf{a}\neq 0$$

Taking Proposition 3 into account we can write

$$\mathbf{G}_0^{\sim}(\xi')=-(2\pi)^2\Delta^{\sim}(\xi')\,\mathbf{V}^{\sim}(\xi') \tag{3.5}$$

where $\mathbf{V}^{\vee}\in S^{-1}$ is the principal symbol of the integral operator with a weak (integrable) singularity on $\partial\Omega$, while $\Delta^{\sim}$ is the principal symbol of the Beltrami–Laplace operator on $\partial\Omega$. An inverse Fourier transformation applied to (3.5) gives

$$\mathbf{G}_0=\mathbf{V}\circ\Delta+\mathbf{r} \tag{3.6}$$

where $\mathbf{r}$ is an operator of class $S^{\circ}$ on $\partial\Omega$.

*Proposition 4.* The calculation of the supersingular integral with kernel $\mathbf{G}_0$ of the function $\mathbf{g}\in H^s(\partial\Omega,R^3)$ with the elimination of the neighbourhood of the pole $\omega_\epsilon$ gives an error $O(\mathrm{mes}(\omega_\epsilon))$.

*Proof.* The right-hand side (0.3) shows that $\mathbf{G}_0(\mathbf{g})(\mathbf{x}')=\mathbf{G}_0(\mathbf{g}-\mathbf{g}_{x'})(\mathbf{x}')$, where $\mathbf{g}_{x'}$ is a constant vector field on $\partial\Omega$, equal to the value of $\mathbf{g}$ at the point $\mathbf{x}'$. Hence, the principal part of the operator $\mathbf{r}$ in (3.6) is a

Calderon–Zygmund operator, and does not contain $\delta$-components. Since $\mathbf{g}(\mathbf{y}') - \mathbf{g}_{x'}|_{y'=x'} = 0$, in the assumption that the field $\mathbf{g}$ is locally constant in the neighbourhood of $\mathbf{x}'$, the supersingular integral $\mathbf{G}_0(\mathbf{g}-\mathbf{g}_{x'})$ turns out to be correctly defined as an integral in the sense of the principal value.

Hence, to determine the values of the supersingular operator $\mathbf{G}_0(\mathbf{g})$ one can use standard programs for evaluating singular integrals with a weak singularity based on ignoring regions containing non-integrable singularities.

## 4. DETERMINATION OF THE KERNEL $\mathbf{G}_0$ BY THE METHOD OF MULTIPOLAR EXPANSIONS

The method of analysing the operator $\mathbf{G}_0$ used in the previous section, based on an investigation of the corresponding symbol, is not very convenient for constructing this operator in practice, particularly for problems with an arbitrary elastic anisotropy.

Below, we construct the kernel of the operator $\mathbf{G}_0$ by the method of multipole expansions. This method was used to establish fundamental solutions of the equations of equilibrium from their symbols in [4] and to establish operators [9] similar to those considered below.

We define the symbol $Z_0^\vee$ as follows:

$$\mathbf{Z}_0^\vee(\boldsymbol{\xi}) = \mathbf{C}\cdot\cdot\,\boldsymbol{\xi}\otimes\mathbf{E}^\vee(\boldsymbol{\xi})\otimes\boldsymbol{\xi}\cdot\cdot\,\mathbf{C} \tag{4.1}$$

By convolution with the vectors of the unit normals $\nu_{x'}$, $\nu_{y'}$, $\mathbf{x}'$, $\mathbf{y}' \in \partial\Omega$ from (4.1) we obtain the amplitude

$$\mathbf{G}_0^\vee(\boldsymbol{\xi}) = \mathbf{T}^\vee(\nu_{x'},\boldsymbol{\xi})\cdot\mathbf{E}^\vee(\boldsymbol{\xi})\cdot\mathbf{T}^{\vee t}(\nu_{y'},\boldsymbol{\xi}) = \nu_{x'}\cdot\mathbf{Z}_0^\vee(\boldsymbol{\xi})\cdot\nu_{y'} \tag{4.2}$$

It in turn generates the symbol $\mathbf{G}_0^\sim$, which was investigated in the previous section.

We will consider the expansion of the symbol $\mathbf{Z}_0^\vee$ which is positively homogeneous of degree zero in the multipole series (the series in surface spherical harmonics)

$$\mathbf{Z}_0^\vee(\boldsymbol{\xi}') = \sum_{p=0,2,\ldots}^{\infty}\sum_{k=1}^{2p+1}\mathbf{Z}^{p,k}Y_k^p(\boldsymbol{\xi}') \tag{4.3}$$

$$\mathbf{Z}^{p,k} = (2\pi)^{-2}\int_S \mathbf{Z}_0^\vee(\boldsymbol{\xi}')\,Y_k^p(\boldsymbol{\xi}')\,d\boldsymbol{\xi}'$$

where $Y_k^p$ are spherical harmonics, while the tensor coefficients $\mathbf{Z}^{p,k}$ are found by integration over the sphere $S$ of unit radius in $R^3$.

The fact that in expansion (4.3) there are only harmonics of even power, is due to the positive homogeneity of the symbol $\mathbf{Z}_0^\vee$. An inverse Fourier transformation of (4.3) gives [10]

$$\mathbf{Z}_0'(\mathbf{x}) = \pi^{-3/2}\sum_{p=2,4,\ldots}^{\infty}(-1)^{p/2}\frac{\Gamma((p+3)/2)}{\Gamma(p/2)}\sum_{k=1}^{2p+1}\mathbf{Z}^{p,k}\frac{Y_k^p(\mathbf{x}')}{|\mathbf{x}|^3} \tag{4.4}$$

In this formula we have omitted the term corresponding to the spherical harmonic of zero degree (a constant) in (4.3), which, in the inverse Fourier transformation, would lead to the occurrence of a $\delta$-like component. The latter, as can easily be seen, disappears for contractions on a manifold of less dimensions and, in particular, on $\partial\Omega$.

Hence, series (4.4) defines the supersingular kernel $\mathbf{Z}_0'$ on $\partial\Omega$. Carrying out the convolution with the vectors $\boldsymbol{\nu}_{x'}$, $\boldsymbol{\nu}_{y'}$, we obtain the required kernel of the operator $\mathbf{G}_0$. The fundamental problems of the convergence and the numerical realization of the method of multipole expansions were considered in [4].

*Note 2.* In addition to multipole expansions, one can also establish the operator $\mathbf{Z}_0'$ from the corresponding symbol for arbitrary anisotropy of the medium using a Radon transformation, for the class of problems considered which is a disintegration of Lebesgue measure over planes in an inverse Fourier transformation. This method is also called the method of expansion in plane waves [11]. Numerical experiments on the inversion of symbols of the fundamental solutions have shown [12] that, in the general case of anisotropy, this method involves considerable computer costs and, for effective realization, requires additional approximations on spheres—essentially expansions of the characteristics or symbols $\mathbf{E}^{\vee}$ in multipoles. Similar complications arise in the Radon method when establishing the operator $\mathbf{Z}_0'$.

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